Preprint typeset in JHEP style - PAPER VERSION

A Consistency Relation for Power Law Inflation in DBI models

Michał Spaliński*

Sołtan Institute for Nuclear Studies
ul. Hoża 69, 00-681 Warszawa, Polska.

ABSTRACT: Brane inflation in string theory leads to a new realization of power law inflation which can give rise to significant non-gaussianity. This can happen for any throat geometry if the scalar potential is appropriate. This note presents a consistency relation connecting the running of the nonlinearity parameter characterizing the non-gaussianity and the scalar and tensor indices. The relationship is valid assuming that the throat geometry and scalar potential support power law inflation, regardless of the level of non-gaussianity.

KEYWORDS: String theory; Cosmology.

*Email: mspal@fuw.edu.pl

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1. Introduction

Given the importance of inflation in our current view of cosmology it is natural and important to try to understand the details of it in the framework of string theory. From the string theory perspective general relativity is a low energy effective field theory, which receives corrections both at the classical and quantum level. These corrections may be crucial in the very early stages of evolution of our Universe. String theory should also determine the degrees of freedom relevant at the time inflation is expected to occur; specifically, it should provide an inflaton.

One possible scenario is brane inflation [1]–[8], which interprets inflation as the motion of a $D3$-brane down a throat in a warped Calabi-Yau compactification [9]. Brane inflation has a rather distinct character because the inflaton is identified with a brane position, and the relevant scalar field kinetic energy functional is non-canonical. Its form is determined by T-duality to be a Dirac-Born-Infeld action [10]. When restricted to spatially homogeneous configurations the action of the inflaton reduces to a DBI scalar field theory studied in a number of papers over the past few years [11]–[18]. The difference between the DBI action and a canonical scalar field action may be interpreted as a classical correction coming from string theory.

It is clearly very important to try to determine observational possibilities which could distinguish this scenario from other options. From the point of view of comparing inflationary models to observation there are a number of properties which significantly restrict the spectrum of possibilities. In this context the most important quantities are inflationary observables like the scalar and tensor spectral indices and the primordial non-gaussianity, as well as the running of these quantities.

One of the interesting features of DBI inflation is the natural appearance of significant levels of primordial non-gaussianity in the spectrum of curvature perturbations. This is a direct consequence of the nonlinear corrections to the scalar kinetic terms. Furthermore, as Chen has emphasized [19], deviations from scale invariance responsible for the running of the scalar and tensor spectral indices also induce running of the non-gaussianity.

In an interesting recent paper on brane inflation [13] Lidsey and Seery have derived a rather general relation involving the nonlinearity parameter often used to describe primordial non-gaussianity in certain simple kinematical configurations. This note applies the same approach to power law inflation in DBI scalar field theories. The result is a relation between the inflationary observables involving the running of the nonlinearity parameter. The observational prospects for actually measuring this parameter are still somewhat remote, but not hopeless.

2. Inflationary Observables

Inflationary observables related to the primordial perturbation spectra have been calculated (to leading order in the Hubble slow roll parameters) by Garriga and Mukhanov [20] for a wide class of scalar field theories, which can be described by the action

$$S = \int d^4x \sqrt{-g}(R + P(X, \phi)) \ , \tag{2.1}$$

where $X \equiv -\frac{1}{2}(\partial\phi)^2$. Their results can be written in terms of Hubble slow roll parameters

$$\epsilon_H = -\frac{1}{H}\frac{d}{dt}\ln H \tag{2.2}$$
$$\eta_H = -\frac{1}{H}\frac{d}{dt}\ln \epsilon_H \tag{2.3}$$
$$\sigma_H = -\frac{1}{H}\frac{d}{dt}\ln c_s\ , \tag{2.4}$$

where $c_s = \partial p/\partial\rho$ is the speed of sound:

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}\ . \tag{2.5}$$

The spectral indices are then given by

$$n_S - 1 = -2\epsilon_H + \eta_H + \sigma_H \tag{2.6}$$
$$n_T = -2\epsilon_H\quad . \tag{2.7}$$

These expressions are valid in the leading order in Hubble slow roll parameters (2.2)-(2.3), which are assumed to be small during the observable phase of inflation.

For the sequel one also needs to recall the notion of the "nonlinearity" parameter f_{NL}, which is an often used measure of non-gaussianity[1]. A simple and explicit formula, valid in a wide range of scalar field theories defined by (2.1), has recently been obtained in [15]:

$$f_{NL} = \frac{35}{108}(\frac{1}{c_s^2} - 1) - \frac{5}{81}(\frac{1}{c_s^2} - 1 - 2\Lambda) \tag{2.8}$$

where

$$\Lambda \equiv \frac{X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}}{XP,X + 2X^2P,XX}\ . \tag{2.9}$$

As emphasized by Chen [19], deviations from scale invariance should manifest themselves also in the running of the non-gaussianity. A measure of it is the index

$$n_{NL} \equiv \frac{d\ln f_{NL}}{d\ln k} \tag{2.10}$$

defined in [19].

[1]See [15] for the precise definition.

3. DBI scalar field theories

The inflaton in brane inflation scenarios is an open string mode, which implies that its dynamics are described by the Dirac-Born-Infeld action. For spatially homogeneous inflaton configurations the action takes the form [11, 4]

$$S = -\int d^4x \; a(t)^3 \; \{f(\phi)^{-1}(\sqrt{1-f(\phi)\dot{\phi}^2}-1)+V(\phi)\} \; . \tag{3.1}$$

The function f appearing here can be expressed in terms of the warp factor in the metric and the $D3$-brane tension.

It is convenient to use the Hamilton-Jacobi formalism [21]–[24], which makes use of the Hubble parameter expressed as function of the scalar field[2]. The basic point is to eliminate the field derivative using the relation

$$\dot{\phi} = -\frac{2M_P^2}{\gamma}H' \; , \tag{3.2}$$

where γ is given as a function of ϕ by

$$\gamma(\phi) = \sqrt{1+4M_P^4 f(\phi)H'(\phi)^2} \; . \tag{3.3}$$

Using this one can calculate the Hubble slow roll parameters

$$\epsilon_H = \frac{2M_P^2}{\gamma}(\frac{H'}{H})^2 \tag{3.4}$$

$$\sigma_H = -\frac{2M_P^2}{\gamma}\frac{H'}{H}\gamma' \tag{3.5}$$

$$\eta_H = \frac{4M_P^2}{\gamma}\frac{H''}{H} - 2\epsilon_H + \sigma_H \; . \tag{3.6}$$

The formula for the nonlinearity parameter (2.8) in the present case simplifies, since one has [15] $c_s = \gamma^{-1}$ and $\Lambda = 0$. This leads to the simple result [12], [15]:

$$f_{NL} = \frac{35}{108}(\gamma^2-1) \; . \tag{3.7}$$

This shows that non-gaussianity in DBI models becomes large in the “ultra-relativistic” regime $\gamma \gg 1$ [12].

[2] In the context of DBI scalar field theories the Hamilton-Jacobi formalism was introduced in [11] and was recently discussed in [17].

Lidsey and Seery [13] have noted that using (2.7) and (3.7) one can turn the expression for the tensor to scalar ratio

$$r = \frac{16\epsilon_H}{\gamma} \tag{3.8}$$

into a consistency relation involving only observable parameters:

$$8n_T = -r\sqrt{1 + \frac{108}{35}f_{NL}}\ , \tag{3.9}$$

which is valid for any DBI scalar field theory, and generalizes the usual consistency relation appearing in [26]. It is a very interesting, testable, prediction of the brane inflation scenario.

The authors of [13] also considered a special case of inflation near the bottom of a warped throat [7] to derive further relations between observable parameters in that situation. In a similar spirit, the following section turns to power law inflation in DBI scalar field theories, where one can obtain another consistency relation of this type, involving the running non-gaussianity parameter (2.10), which can easily be calculated in this class of models:

$$n_{NL} = -4M_P^2(1 + \frac{35}{108}\frac{1}{f_{NL}})\frac{\gamma'}{\gamma}\frac{H'}{H} \tag{3.10}$$

This is valid to leading order in the Hubble slow roll parameters. As explained in the following section, if one assumes power law inflation then using (3.10) it is possible to rewrite (2.6) as another consistency relation.

4. Power Law Inflation

It was found by Silverstein and Tong [11] that for the case of an AdS throat (where $f(\phi) = \lambda/\phi^4$) a quadratic potential with a suitably high inflaton mass leads to power law inflation in the "ultra-relativistic" regime $\gamma \gg 1$. It was subsequently pointed out that power law inflationary solutions exist in DBI scalar field theories even when γ is not large [18]. Furthermore, for any throat geometry there is a potential which leads to power law inflation for some range of parameters. This generalizes the well known fact that in the case of canonical kinetic terms exponential potentials lead to power law inflation [25].

Power law inflation occurs when the ratio of pressure over energy density is constant, i.e. when the parameter w in the baryotropic equation of state $p = w\rho$ is constant and $w < -1/3$. As shown in [18], power law inflationary solutions will exist if the potential is of the form

$$V(\phi) = 3M_P^2 H(\phi)^2 - \frac{\gamma(\phi) - 1}{f(\phi)} \ , \tag{4.1}$$

where $H(\phi)$ satisfies the differential equation

$$4M_P^2 H'^2 = 3(w+1)H^2\sqrt{1 + 4M_P^4 f H'^2} \tag{4.2}$$

with $w < -1/3$.

The essential property of power law inflation is that the parameter ϵ_H is constant. Indeed, from (3.4) and (4.2) one concludes that

$$\epsilon_H = \frac{3}{2}(w+1). \tag{4.3}$$

One immediate consequence is that the tensor spectral index does not run, since by virtue of (2.7) it is constant. Furthermore, a measurement of the tensor spectral index would determine the parameter w, which in the model of [11] is related to the inflaton mass [18].

Since ϵ_H is constant it also follows that η_H (defined in (2.3)) vanishes[3]. This makes it possible to derive another consistency relation involving the spectral indices, the non-gaussianity, and running of the nonlinearity parameter (2.10). Indeed, from (3.5) and (3.10) it follows that

$$\sigma_H = \frac{1}{2} n_{NL} (1 + \frac{35}{108}\frac{1}{f_{NL}})^{-1} \ . \tag{4.4}$$

Using this and $\eta_H = 0$, (2.6) takes the form

$$n_S - n_T = 1 + \frac{1}{2} n_{NL} (1 + \frac{35}{108}\frac{1}{f_{NL}})^{-1} \ , \tag{4.5}$$

which is a relation between observable parameters. It is valid for any DBI scalar field theory solution describing power law inflation. In particular, it does not assume

[3]Some authors (e.g. [4]) define a different "η" parameter in this context, related to η_H by $\eta_D = \eta_H + 2\epsilon_H - \sigma_H$. In that language power law inflation implies the relation $\eta_D = 2\epsilon_H - \sigma_H$.

simplifications which occur in the "ultra-relativistic" limit, so one can also consider the case of f_{NL} small or zero in this expression.

In the "ulrarelativistic" limit, when the non-gaussianity is large, this relation can be further simplified to

$$n_S - n_T = 1 + \frac{1}{2} n_{NL} \ . \tag{4.6}$$

While the prospect of measuring n_{NL} may seem distant today, these relations may be tested at some point in the future.

5. Conclusions

Single field inflation with canonical kinetic energy terms leads to negligible non-gaussianity [28], [29]. While it is too early to tell whether observation will require more general models of inflation, a lot of attention has been devoted to models where large non-gaussianity may naturally occur. One possibility is models with multiple scalars [27]. Another option is DBI inflation, which can generate significant non-gaussianity during a power law inflationary stage.

In field theoretical models power law inflation is realized by an exponential scalar potential, so there is no natural mechanism for inflation to end. One needs to supplement the exponential potential by some external agent which terminates inflation. In the context of brane inflation this role is played by a tachyon which appears when the mobile $D3$-brane gets within a warped string length of the anti-brane at the bottom of the throat. The process of brane annihilation ends inflation and the energy released is (hopefully [30], [31]) transferred to standard-model degrees of freedom localized in another throat in the compactification manifold. One may thus argue that power law inflation finds a very natural place in the brane inflation scheme.

The consistency relation (4.5) is a consequence of assuming power law inflation, but it is valid in DBI scalar field theories without necessarily assuming the "ultra-relativistic" limit $\gamma \gg 1$. It is also worth stressing that it is not restricted to the specific realization of power law inflation discussed in [11], i.e. a quadratic potential and an anti-de-Sitter throat. This is rather important, in that there are many contributions to the scalar potential, which are at the moment hard to control. There are also various possibilities for warped throats in type IIB compactifications, and different opinions as to which section of the throat is relevant for inflation [7], as well as to the direction of the D-brane motion [5], [6]. The consistency relation derived here does not assume a specific choice in these matters; it should be valid whenever the resulting inflationary stage has power law character.

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